3/9/18 "Do what is right, not what is easy"-Unknown

HW: "Forms of a Line" Homework section
Test 2 on Wednesday 3/14

AIM: What are th different forms of a linear equation?

Warm Up:

***Exercise* #1:** Consider the linear function $f(x) = 3x + 5$.

(a) Determine the y-intercept of this function by evaluating $f(0)$.

$f(0) = 3(0) + 5$

$f(0) = 5$

(b) Find its $\underbrace{\text{average rate of change}}_{\text{slope}}$ over the interval $-2 \le x \le 3$.

$(-2, -1)$ $(3, 14)$

$$\frac{\Delta y}{\Delta x} = \frac{14-(-1)}{3-(-2)} = \frac{15}{5} = 3$$

COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. The sum of two numbers is 5 and the larger difference of the two numbers is 39. Find the two numbers by setting up a system of two equations with two unknowns and solving algebraically.

$$\begin{aligned} x+y&=5 \\ x-y&=39 \end{aligned} \Rightarrow \begin{aligned} x+y&=5 \\ x-y&=39 \\ \hline 2x&=44 \\ x&=22 \end{aligned} \Rightarrow \begin{aligned} 22+y&=5 \\ y&=-17 \end{aligned} \Rightarrow$$

The two numbers are: −17 and 22

2. Algebraically, find the intersection points of the two lines whose equations are shown below.

$$4x+3y=-13$$
$$y=6x-8$$

$$\begin{aligned} 4x+3(6x-8)&=-13 \\ 4x+18x-24&=-13 \\ 22x-24&=-13 \\ 22x&=11 \\ x&=\frac{11}{22}=\frac{1}{2} \end{aligned} \Rightarrow \begin{aligned} y&=6x-8 \\ y&=6\left(\frac{1}{2}\right)-8 \\ y&=3-8 \\ y&=-5 \end{aligned} \Rightarrow$$

Intersection point: $\left(\frac{1}{2},-5\right)$

3. Show that $x=10,\ y=4,$ and $z=7$ is a solution to the system below *without* solving the system formally.

$$x+2y+z=25$$
$$4x-y-5z=1$$
$$-2x-y+8z=32$$

We simply need to show that each equation is true when these values of *x*, *y*, and *z* are substituted.

Equation (1):
$$\begin{aligned} 10+2(4)+7&=25 \\ 10+8+7&=25 \\ 25&=25 \end{aligned}$$

Equation (2):
$$\begin{aligned} 4(10)-4-5(7)&=1 \\ 40-4-35&=1 \\ 1&=1 \end{aligned}$$

Equation (3):
$$\begin{aligned} -2(10)-4+8(7)&=32 \\ -20-4+56&=32 \\ 32&=32 \end{aligned}$$

4. In the following system, the value of the constant c is unknown, but it is known that $x=-8$ and $y=4$ are the x and y values that solve this system. Determine the value of c. Show how you arrived at your answer.

$$-5x+2y+3z=81$$
$$x-y+z=-1$$
$$2x-y+cz=35$$

We first need to know the *z*-value that solves the system. We can use either the first or second equation to find it. We choose the second equation.

$$\begin{aligned} -8-4+z&=-1 \\ z-12&=-1 \\ z&=11 \end{aligned} \Rightarrow \begin{aligned} 2(-8)-4+c(11)&=35 \\ 11c-20&=35 \\ 11c&=55 \\ c&=5 \end{aligned}$$

5. Solve the following system of equations. Carefully show how you arrived at your answers.

$$\begin{aligned} 4x+2y-z&=21\\ -x-2y+2z&=13\\ 3x-2y+5z&=70 \end{aligned}$$

$(1)+(2)$:
$$\begin{aligned} 4x+2y-z&=21\\ -x-2y+2z&=13\\ \hline 4 \quad 3x+z&=34 \end{aligned}$$

$(1)+(3)$:
$$\begin{aligned} 4x+2y-z&=21\\ 3x-2y+5z&=70\\ \hline 5 \quad 7x+4z&=91 \end{aligned}$$

$$\begin{aligned} 3x+z&=34\\ 7x+4z&=91 \end{aligned}$$

$$\begin{aligned} -4(3x+z)&=-4(34)\\ -12x-4z&=-136 \end{aligned}$$

$$\begin{aligned} -12x-4z&=-136\\ 7x+4z&=91\\ \hline -5x&=-45\\ x&=9 \end{aligned}$$

$$\begin{aligned} -12(9)-4z&=-136\\ -108-4z&=-136\\ -4z&=-28\\ z&=7 \end{aligned}$$

$$\begin{aligned} 4(9)+2y-7&=21\\ 2y+29&=21\\ 2y&=-8\\ y&=-4 \end{aligned}$$

6. Algebraically solve the following system of equations. There are two variables that can be readily eliminated, but your answers will be the same no matter which you eliminate first.

$$\begin{aligned} 2x+5y-z&=-35\\ x-3y+4z&=31\\ -3x+2y+2z&=-23 \end{aligned}$$

In this problem we will choose to eliminate the *z*-variable by manipulating equation (1).

$4\times(1)+(2)$:
$$\begin{aligned} 8x+20y-4z&=-140\\ x-3y+4z&=31\\ \hline 9x+17y&=-109 \end{aligned}$$

$2\times(1)+(3)$:
$$\begin{aligned} 4x+10y-2z&=-70\\ -3x+2y+2z&=-23\\ \hline x+12y&=-93 \end{aligned}$$

$$\begin{aligned} 9x+17y&=-109\\ x+12y&=-93 \end{aligned}$$

$$\begin{aligned} -9(x+12y)&=-9(-93)\\ -9x-108y&=837 \end{aligned}$$

$$\begin{aligned} 9x+17y&=-109\\ -9x-108y&=837\\ \hline -91y&=728\\ y&=-8 \end{aligned}$$

$$\begin{aligned} 9x+17(-8)&=-109\\ 9x-136&=-109\\ 9x&=27\\ x&=3 \end{aligned}$$

$$\begin{aligned} 2(3)+5(-8)-z&=-35\\ -z-34&=-35\\ -z&=-1\\ z&=1 \end{aligned}$$

7. Algebraically solve the following system of equations. This system will take more manipulation because there are no variables with coefficients equal to 1.

$$\begin{aligned} 2x+3y-2z&=33\\ 4x+5y+3z&=54\\ -6x-2y-8z&=-50 \end{aligned}$$

Here we choose to eliminate the variable *x* by manipulating equation (1).

$-2\times(1)+(2)$:
$$\begin{aligned} -4x-6y+4z&=-66\\ 4x+5y+3z&=54\\ \hline -y+7z&=-12 \end{aligned}$$

$3\times(1)+(3)$:
$$\begin{aligned} 6x+9y-6z&=99\\ -6x-2y-8z&=-50\\ \hline 7y-14z&=49 \end{aligned}$$

$$\begin{aligned} -y+7z&=-12\\ 7y-14z&=49 \end{aligned}$$

$$\begin{aligned} 2(-y+7z)&=2(-12)\\ -2y+14z&=-24 \end{aligned}$$

$$\begin{aligned} -2y+14z&=-24\\ 7y-14z&=49\\ \hline 5y&=25\\ y&=5 \end{aligned}$$

$$\begin{aligned} -2(5)+14z&=-24\\ 14z-10&=-24\\ 14z&=-14\\ z&=-1 \end{aligned}$$

$$\begin{aligned} 2x+3(5)-2(-1)&=33\\ 2x+17&=33\\ 2x&=16\\ x&=8 \end{aligned}$$

***Exercise* #2:** Consider a line whose slope is 5 and which passes through the point $(-2, 8)$.

(a) Write the equation of this line in point-slope form, $y - y_1 = m(x - x_1)$.

$y - 8 = 5(x - (-2))$

red is always in there.

(b) Write the equation of this line in slope-intercept form, $y = mx + b$.

$y - 8 = 5(x + 2)$

$y - 8 = 5x + 10$

$\quad +8 \qquad\quad +8$

$\boxed{y = 5x + 18}$

***Exercise* #3:** Which of the following represents an equation for the line that is ~~parallel~~ parallel to $y = \frac{3}{2}x - 7$ and which passes through the point $(6, -8)$?

x y

Same slope

(1) $\cancel{y - 8 = -\frac{2}{3}(x+6)}$

(3) $y + 8 = \frac{3}{2}(x - 6)$

(2) $y - 8 = \frac{3}{2}(x+6)$

(4) $\cancel{y + 8 = -\frac{2}{3}(x-6)}$

$y - (-8) = \frac{3}{2}(x - 6)$

***Exercise* #4:** A line passes through the points $(5, -2)$ and $(20, 4)$.

(a) Determine the slope of this line in simplest rational form.

$$\frac{\Delta y}{\Delta x} = \frac{-2-4}{5-20} = \frac{-6}{-15} = \boxed{\frac{2}{5}}$$

(b) Write an equation of this line in point-slope form.

$$y-(-2) = \frac{2}{5}(x-5)$$

OR $y = \frac{2}{5}x - 4$

$$y-4 = \frac{2}{5}(x-20)$$

$$\frac{2}{5}x - 4$$

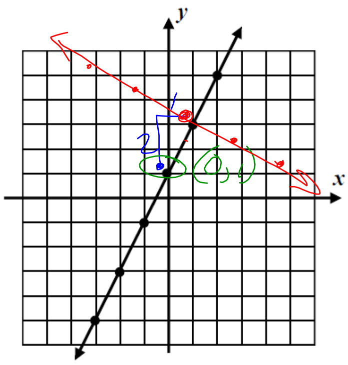
(c) Write an equation for this line in slope-intercept form.

$$y = \frac{2}{5}x - 4$$

(d) For what x-value will this line pass through a y-value of 12?

$$12 = \frac{2}{5}x - 4$$
$$+4 \qquad +4$$
$$\frac{5}{2}\,16 = \cancel{\frac{5}{2}}\cancel{\frac{2}{5}}x$$
$$\boxed{40 = x}$$

***Exercise* #5:** The graph of a linear function is shown below.



(a) Write the equation of this line in $y = mx + b$ form.

$m = \frac{2}{1}$ $\quad \boxed{y = 2x + 1}$ $\quad b = 1$

(b) What must be the slope of a line perpendicular to the one shown?

$m_{\perp} = -\frac{1}{2}$

negative reciprocal slope (flip & switch sign)

(c) Draw a line perpendicular to the one shown that passes through the point $(1, 3)$.

(d) Write the equation of the line you just drew in point-slope form.

$$y-3 = -\frac{1}{2}(x-1)$$

(e) Does the line that you drew contain the point $(30, -15)$? Justify.

$$-15-3 = -\frac{1}{2}(30-1)$$
$$-18 = -\frac{1}{2}(29)$$
$$-18 = -14.5$$
$$\boxed{\text{NO}}$$